Periodicity as a Dynamical Aspect of Generative SN P Systems

Gyenn Neil V. Ibo

Henry N. Adorna

Algorithms and Complexity Laboratory
Dept. of Computer Science
University of the Philippines Diliman

Outline

- Introduction
- Matrix Representation of SN P Systems
- Periodicity in SN P Systems
- Final Remarks
- On-going and Future Work

Peculiarities of SN P Systems

- Single object consisting the Alphabet
- Time as information representation

- Dynamical Aspects of P Systems
  - V. Manca and F. Bernardini (2003)
- With time playing a key role in SN P systems, it is all the more interesting to explore its dynamics.
- One aspect of an SN P system's dynamics of particular interest: Periodicity.

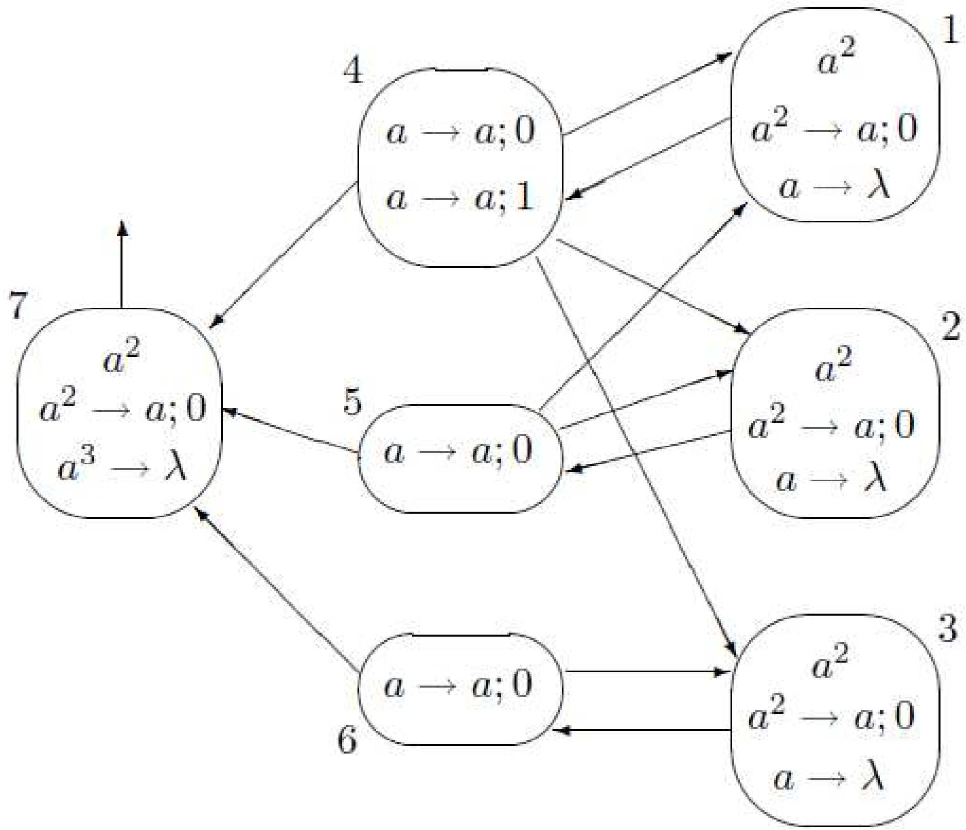


Figure 1. An SN P system generating all even natural numbers, from *Spiking Neural P Systems (Ionescu, Paun, Yokomori, 2006).*

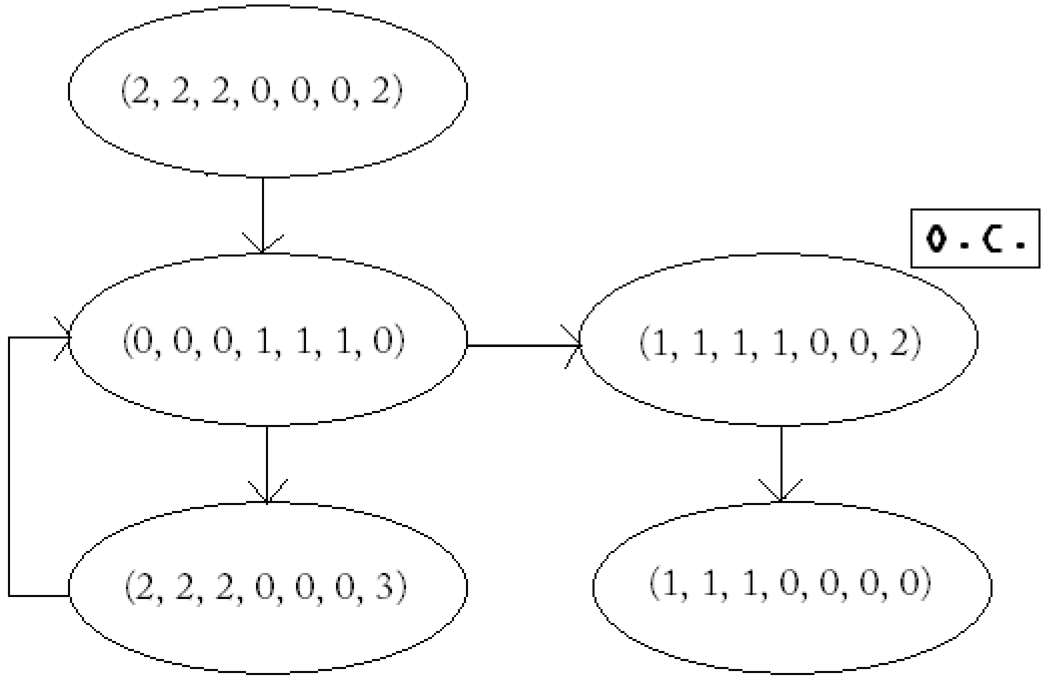


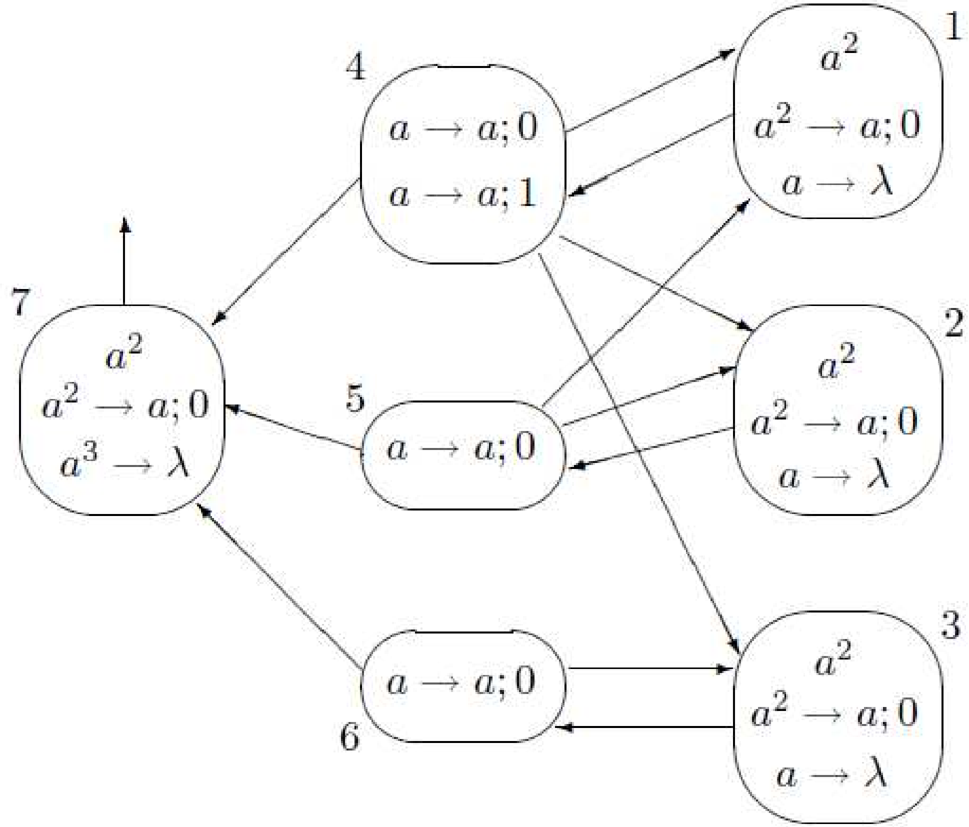
Figure 2. Computation Graph (State Diagram) of the SN P system shown in Figure 1.

Matrix Representation of SN P Systems

(X. Zeng, H. Adorna, M. A. Martínez-del-Amor, L. Pan, M. Pérez-Jiménez)

1. Configuration Vector

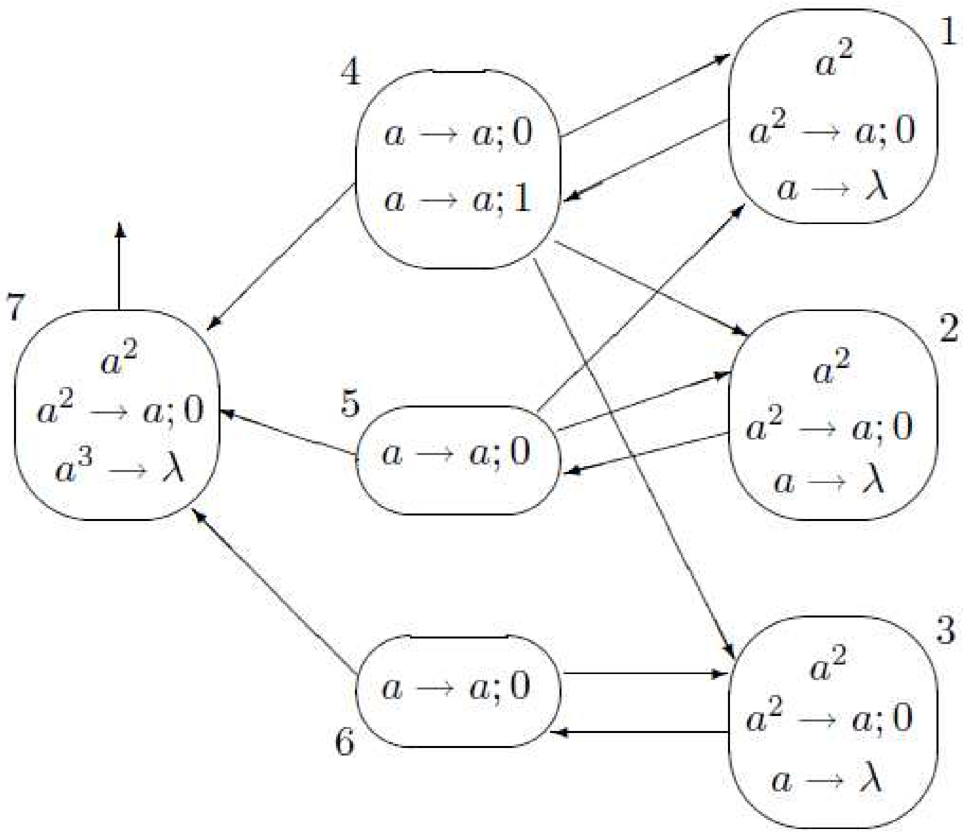
   – represents the number of spikes in the neurons as a vector.


1
a^2
$a^2 \to a;0$
$a \to \lambda$
2
a^2
$a^2 \to a;0$
$a \to \lambda$
3
a^2
$a^2 \to a;0$
$a \to \lambda$
4
$a \to a;0$
$a \to a;1$
5
$a \to a;0$
6
$a \to a;0$
7
a^2
$a^2 \to a;0$
$a^3 \to \lambda$

$C_k = (2, 2, 2, 0, 0, 0, 2)$

2. Spiking Vector

– a vector representing which rules are applied during a particular time-step.



$s^k = (1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0)$

3. Spiking Transition Matrix

$$M_{\Pi} = [a_{ij}]_{nxm},$$

where a_{ij} is equal to the following:

= -c, if rule r_i is in neuron σ_j and it is applied consuming c spikes;

= p, if rule r_i is in neuron $sigma_s$ ($s \neq j$ and $(s, j) \in syn$) and it is applied producing p spikes;

= 0, rule r_i is in neuron σ_s ($s \neq j$ and $(s, j) \notin syn$).

$$M_1 = \begin{bmatrix} -2 & 0 & 0 & 1 & 0 & 0 & 0 \\ - & - & - & - & - & - & - \\ 0 & -2 & 0 & 0 & 1 & 0 & 0 \\ - & - & - & - & - & - & - \\ 0 & 0 & -2 & 0 & 0 & 1 & 0 \\ - & - & - & - & - & - & - \\ 1 & 1 & 1 & -1 & 0 & 0 & 0 \\ - & - & - & - & - & - & - \\ 1 & 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & -3 \end{bmatrix}$$

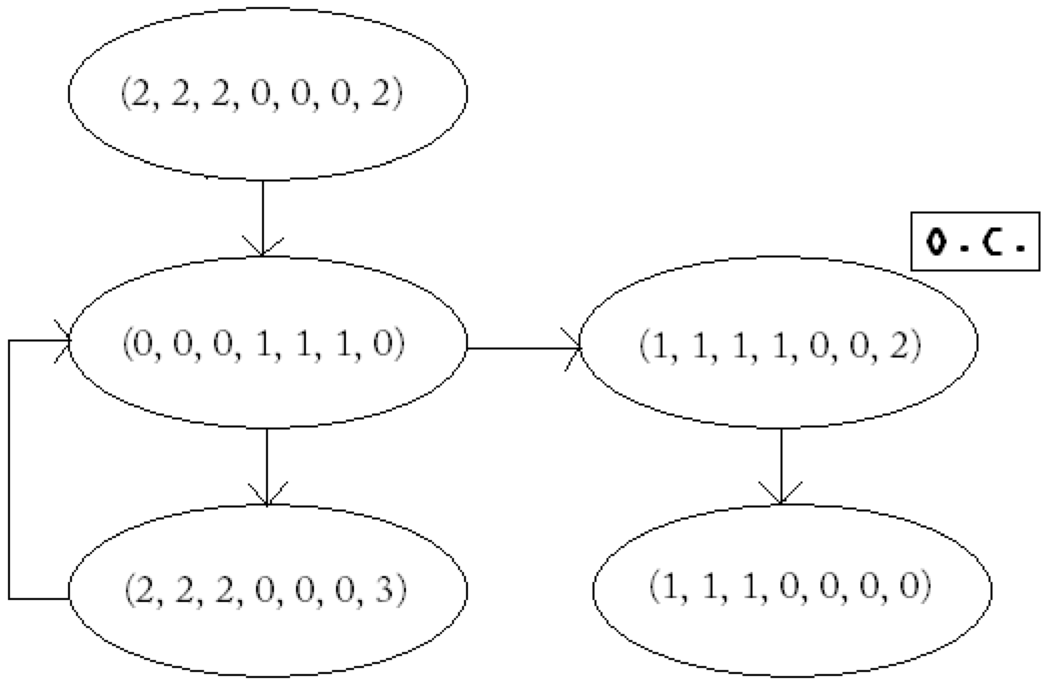
The Spiking Transition Matrix basically represents and stores the information on how each rule affect the number of spikes of each neuron.

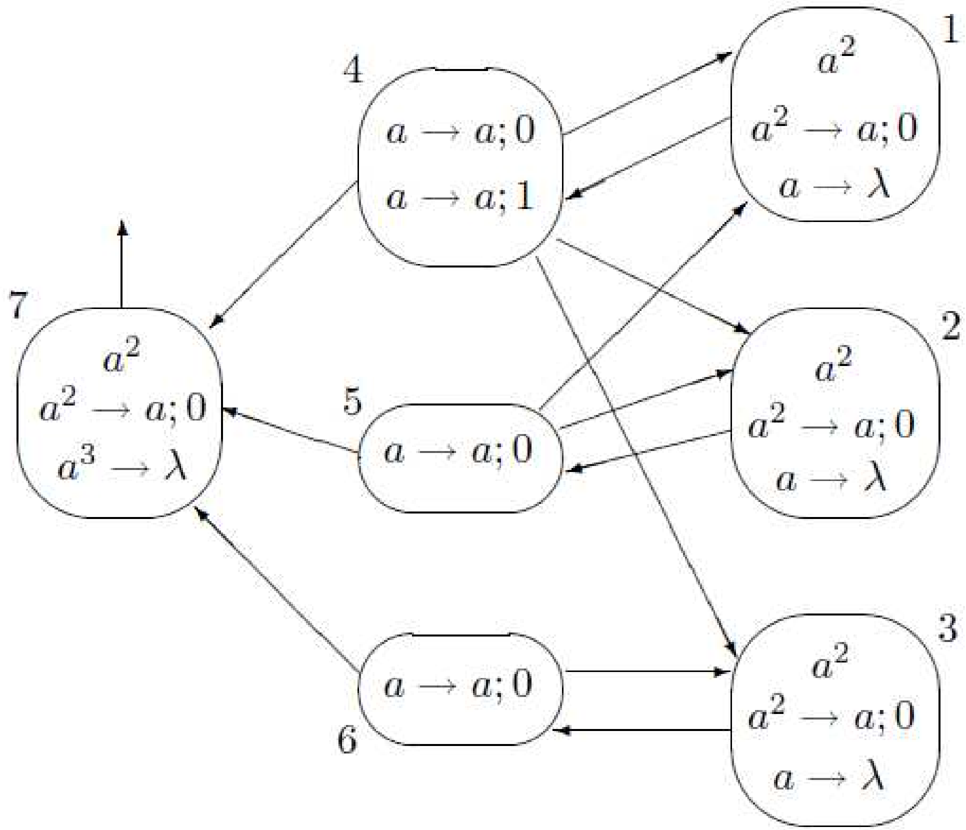
4. Transition Net Gain Vector

- a vector that represents the net amount of spikes to be gained by an SN P system given the rules to be applied
- $NG^k = s_k \cdot M_\pi$
- $NG^k = C_{k+1} – C_k$
- $C_{k+1} = C_k + NG^k$

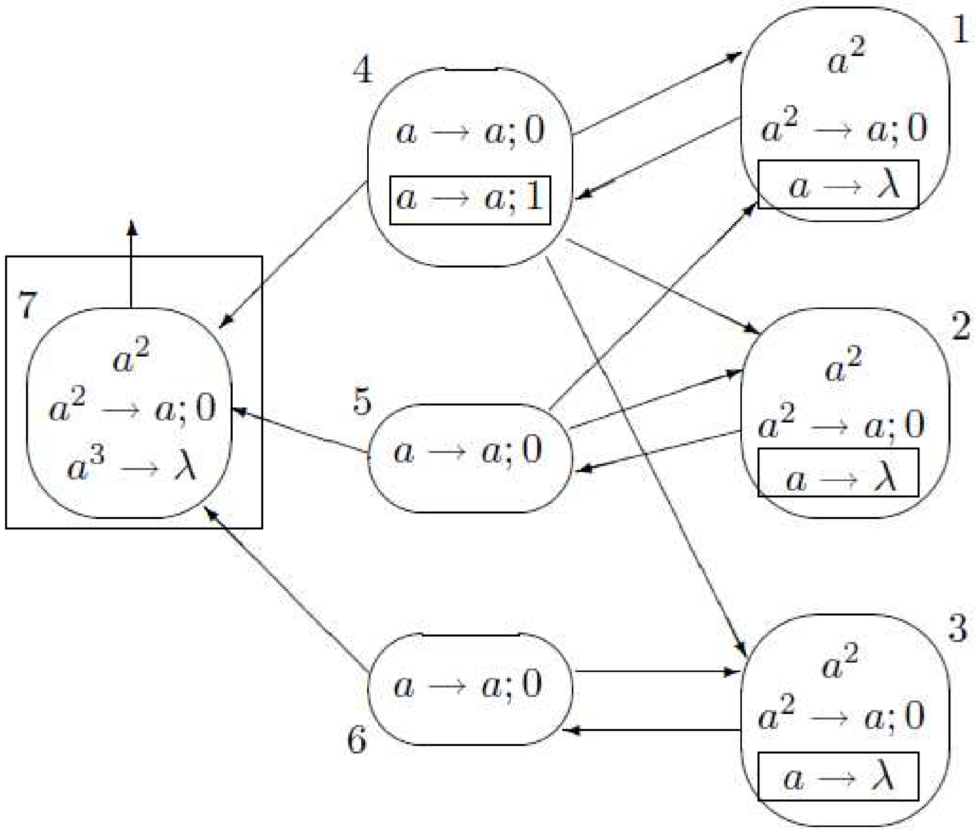
- given: $s^0 = (1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0)$
- $NG^0 = (-2, -2, -2, 1, 1, 1, -2)$

Periodicity in SN P Systems

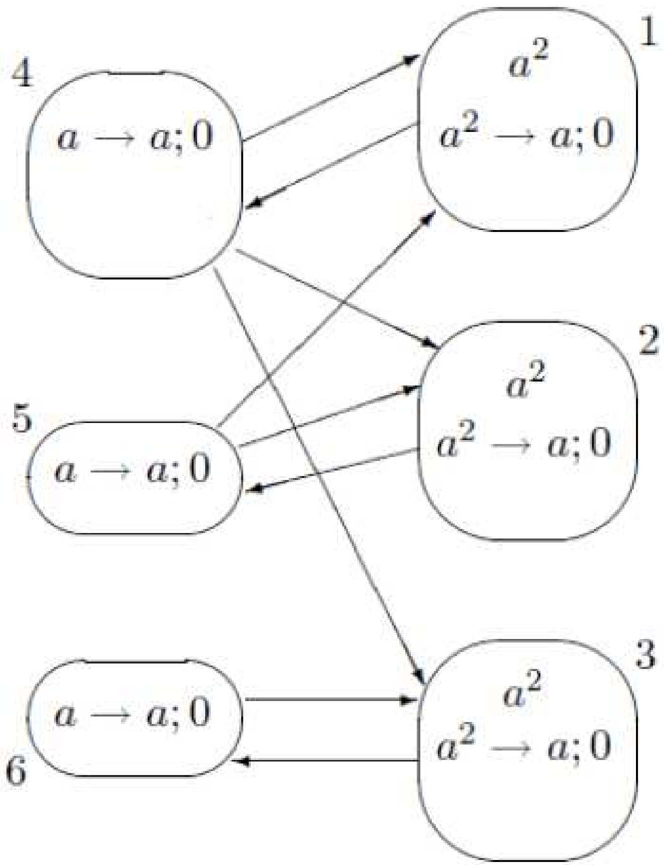




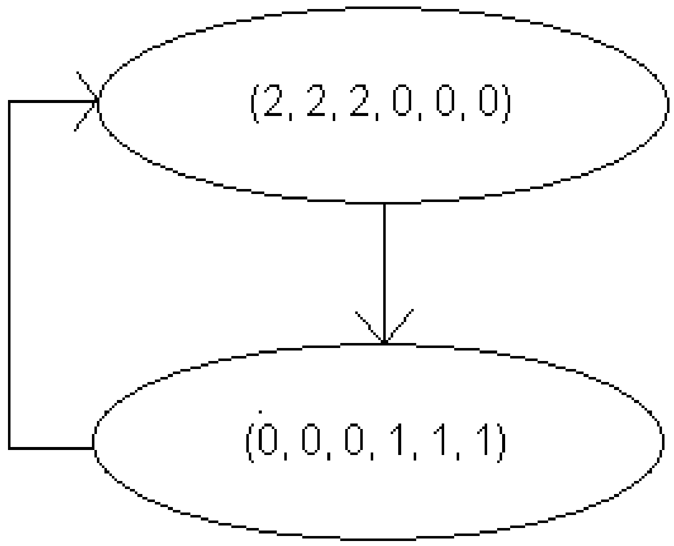
1
a^2
$a^2 \to a; 0$
$a \to \lambda$
2
a^2
$a^2 \to a; 0$
$a \to \lambda$
3
a^2
$a^2 \to a; 0$
$a \to \lambda$
4
$a \to a; 0$
$a \to a; 1$
5
$a \to a; 0$
6
$a \to a; 0$
7
a^2
$a^2 \to a; 0$
$a^3 \to \lambda$



1
a^2
$a^2 \to a; 0$
$a \to \lambda$
2
a^2
$a^2 \to a; 0$
$a \to \lambda$
3
a^2
$a^2 \to a; 0$
$a \to \lambda$
4
$a \to a; 0$
$a \to a; 1$
5
$a \to a; 0$
6
$a \to a; 0$
7
a^2
$a^2 \to a; 0$
$a^3 \to \lambda$



A deterministic, periodic SN P system with period equal to 2.


(2, 2, 2, 0, 0, 0)
(0, 0, 0, 1, 1, 1)

Definitions

Definition (Computation Sequence). *Let Π be an SN P system with m neurons and m rules (one rule for each neuron), $C_0 = (n_1^{(0)}, n_2^{(0)}, ..., n_m^{(0)})$ be the initial configuration vector, $s^{(0)} = (r_1^{(0)}, r_2^{(0)}, ..., r_m^{(0)})$ be the initial spiking vector, and M_Π be the spiking transition matrix of Π. The computation sequence of Π is a sequence $COMP_{seq}$ consists of configuration vectors (as defined in Defintion 2) starting with C_0, with the proceeding elements obtained recursively by the formula:*

$$C_{k+1} = C_k + s^{(k)} \cdot M_\Pi$$

Definition (Periodic SN P system). *Let* Π *be an SN P system with m neurons and m rules (one rule for each neuron), and* $COMP_{seq}$ *be the computation sequence of* Π. *The system* Π *is periodic if and only if there exist configurations* C_k *and* C_p *in* $COMP_{seq}$, $k \neq p$, *and* $C_k = C_p$. *That is, an SN P System is periodic (or ultimately periodic) if and only if one or more configurations are repeated in the sequence* $COMP_{seq}$.

Definition (Spiking Vector Sequence). *Let Π be an SN P system with m neurons and m rules (one rule for each neuron), and $COMP_{seq} = (C_0, C_1, C_2, ...)$ be the computation sequence (where each $C_k = (n_1^{(k)}, n_2^{(k)}, , n_m^{(k)})$) of Π. The spiking vector sequence SV_{seq} is a sequence that consists of spiking vectors $s^{(k)}$, where $s^{(k)} = (r_1^{(k)}, r_2^{(k)}, ..., r_m^{(k)})$ is the spiking vector corresponding to every configuration C_k in the $COMP_{seq}$. This is the sequence of all the spikings during the lifespan of the system Π.*

Note: the spiking vectors $s^{(k)}$ is computed as follows:

$r_i^{(k)}$ is assigned as 1 if the amount of spikes $n_i^{(k)}$ in neuron i in at time-step k satisfies the regular expression E_i; 0 otherwise.

Definition **(Aggregate Spiking Vector).** *Let Π be an SN P system with m neurons and m rules (one rule for each neuron),, and $SV_{seq} = (s^{(0)}, s^{(1)}, s^{(2)}, ...)$ be the spiking vector sequence of Π. The aggregate spiking vector $(SV)_k^{k+v}$ is a summation of any finite consecutive subsequence of SV_{seq}, from the kth to the (k+v)th element.*

Given $(s^{(k)}, s^{(k+1)}, ..., s^{(k+v)})$ as a finite consecutive subsequence of SV_{seq}, the aggregate spiking vector $(SV)_k^{k+v}$ is given by the summation $\sum_{i=k}^{k+v} s^{(i)}$.

Definition **(Transition Net Gain Vector Sequence).** *Let* Π *be an SN P system with* m *neurons and* m *rules (one rule for each neuron) , and* $SV_{seq} = (s^{(0)}, s^{(1)}, s^{(2)}, ...)$ *be the spiking vector sequence of* Π. *The transition net gain vector sequence* NG_{seq} *is a sequence that consists of transition net gain vectors* $NG^{(k)}$, *obtained by the formula* $NG^{(k)} = s^{(k)} \cdot M_{\Pi}$ *(from Equationn 2) for every* $s^{(k)}$ *in the spiking vector sequence* SV_{seq}. *This is the sequence of all the transition net gain vectors during the lifespan of the system* Π.

Definition (Aggregate Transition Net Gain Vector). *Let Π be an SN P system with m neurons and m rules (one rule for each neuron), and $NG_{seq} = (NG^{(0)}, NG^{(1)}, NG^{(2)}, ...)$ be the transition net gain vector sequence of Π. The aggregate transition net gain vector $(NG)_k^{k+v}$ is a summation of any finite consecutive subsequence of NG_{seq}, from the kth to the (k+v)th element.*

Given $(NG^{(k)}, NG^{(k+1)}, ..., NG^{(k+v)})$ as a finite consecutive subsequence of NG_{seq}, the aggregate transition net gain vector $(NG)_k^{k+v}$ is given by the summation $\sum_{i=k}^{k+v} NG^{(i)}$.

Recall the following two equations:

$$NG^k = s_k \cdot M_\pi \qquad [1]$$

$$C_{k+1} = C_k + NG^k \qquad [2]$$

We generalize them into the realm of sequences of vectors, arriving at the following:

$$(NG)_k^{k+v} = (SV)_k^{k+v} \cdot M_\pi \qquad [3]$$

$$C_{k+v} = C_k + (NG)_k^{k+v} \qquad [4]$$

Definition **(Zero-Aggregate Transition Net Gain Vector).** *Let Π be an SN P system with m neurons and m rules (one rule for each neuron), and* $NG_{seq} = (NG^{(0)}, NG^{(1)}, NG^{(2)}, ...)$ *be the transition net gain vector sequence of* Π. *If any aggregate transition net gain vector* $(NG)_k^{k+v}$ *sums up to zero, represented by the vector* $(0^{(1)}, 0^{(2)}, ..., 0^{(m)})$, *we call it the zero-aggregate transition net gain vector.*

Definition (Null Aggregate Spiking Vector). *Let Π be an SN P system with m neurons and m rules (one rule for each neuron), $SV_{seq} = (s^{(0)}, s^{(1)}, s^{(2)}, ...)$ be the spiking vector sequence, and $(SV)_k^{k+v}$ be the aggregate spiking vector during the kth. . .(k+v)th computation steps of Π. If the aggregate transition net gain vector $(NG)_k^{k+v}$, as computed by the formula $(NG)_k^{k+v} = (SV)_k^{k+v} \cdot M_\Pi$ (Equation 10 from Lemma 1) is equal to zero (a zero-aggregate transition net gain vector), then we call $(SV)_k^{k+v}$ a null aggregate spiking vector.*

Lemma *Let Π be an SN P system with m neurons and m rules (one rule for each neuron), M_{Π} be the spiking transition matrix, and SV_{seq} be the spiking vector sequence of Π. The system is periodic if and only if there exist a Null Aggregate Spiking Vector $(SV)_k^{k+v}$ from a finite consecutive subsequence of SV_{seq} which results to a Zero-Aggregate Transition Net Gain Vector (i.e., $(NG)_k^{k+v} = 0$).*

Proof:

- Let C_k be the configuration of the system π, and $s^{(k)}$ be the spiking vector during time step k.

- If the A***ggregate Spiking Vector*** $(SV)_k^{k+v}$ during the time-steps $k, k+1, \ldots, k+v$ is a ***Null Aggregate Spiking Vector***, resulting to a ***Zero-Aggregate Transition Net Gain Vector*** (i.e.,$(NG)_k^{k+v} = 0$), then the configuration C_{k+v}, using Equationn [4] above, is obtained as:

$$C_{k+v} = C_k + (NG)_k^{k+v}$$
$$C_{k+v} = C_k + 0$$
$$C_{k+v} = C_k$$

- Thus, by the aforementioned definition of Periodicity in SN P systems, the system π must be Periodic.

- Conversely, given a periodic system π, then it implies that at least one configuration, say C_k, is repeated after a particular amount of time-steps, say v, where $C_k = C_{k+v}$.

- Likewise, from Equation [4]:

$$C_{k+v} = C_k + (NG)_k^{k+v}$$
$$(NG)_k^{k+v} = C_{k+v} - C_k$$
$$(NG)_k^{k+v} = 0$$ (a ***Zero-Aggregate Transition Net Gain Vector***)

- This means that there must exist a ***Null Aggregate Spiking Vector*** $(SV)_k^{k+v}$ *that yielded the* ***Zero-Aggregate Transition Net Gain Vector.***

■

The following theorem presents a necessary condition for SN P systems to be Periodic.

We claim that if an SN P system is Periodic, then the determinant of its *Spiking Transition Matrix* M_{π} must be equal to zero.

Theorem:

Let $\boldsymbol{\pi}$ *be an SN P system with m rules and m neurons (one rule for each neuron), and* $\boldsymbol{M_\pi}$ *be the* ***Spiking Transition Matrix*** *of π. If π is periodic, then the determinant of the (m x m) matrix* $\boldsymbol{M_\pi}$ must be equal to **zero**.

Proof:

- If $\boldsymbol{\pi}$ is periodic, then by the abovementioned Lemma, there must exist a ***Null Aggregate Spiking Vector*** $\boldsymbol{(SV)_k^{k+v}}$ that yields a ***Zero-Aggregate Transition Net Gain Vector*** $\boldsymbol{((NG)_k^{k+v} = 0)}$.
- Recall Equation [3] above:

$$(NG)_k^{k+v} = (SV)_k^{k+v} \cdot M_\pi$$

- Since $(NG)_k^{k+v} = 0$, then:

$$(SV)_k^{k+v} \cdot M_\pi = 0 \qquad [5]$$

- Solving for the ***Null Aggregate Spiking Vector*** $\boldsymbol{(SV)_k^{k+v}}$ in Equation [5] is essentially equivalent to finding the ***Null Space*** of the matrix $\boldsymbol{M_\pi}$.

- We know that this equation has at least one solution for $\boldsymbol{(SV)_k^{k+v}}$:

  -the zero vector $(0^{(1)}, 0^{(2)}, \ldots, 0^{(m)})$, *the trivial solution.*

- But we ignore such a solution here because an ***Aggreagate Spiking Vector*** $\boldsymbol{(SV)_k^{k+v}}$ with all components zero implies that the system has not spiked (i.e., no rule has been applied.)

- Furthermore, we know that this equation will have solutions other than the trivial solution if and only if the matrix M_π is singular.

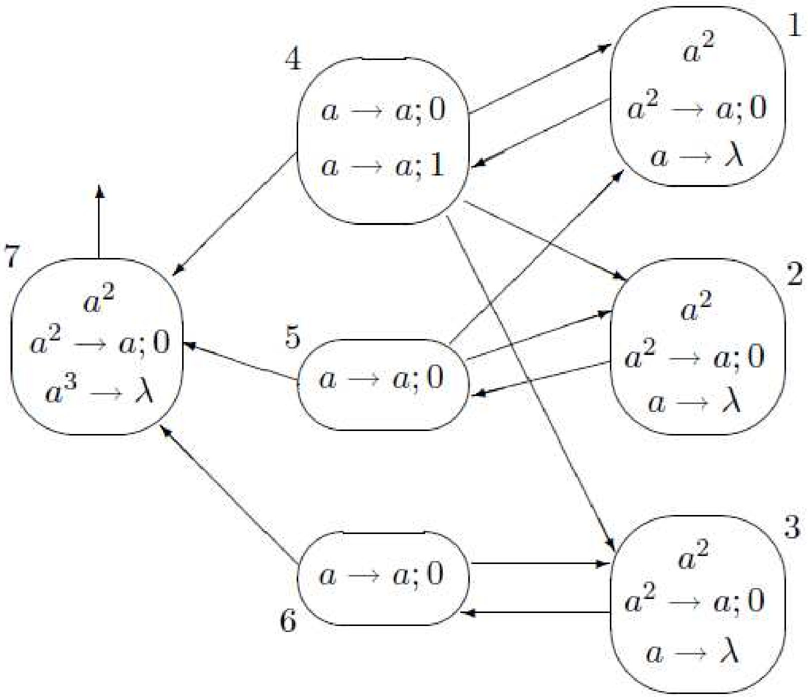
- For square matrices (which we know is true for the *m x m* matrix M_π), if the matrix is singular, then the determinant of the matrix M must be equal to zero.

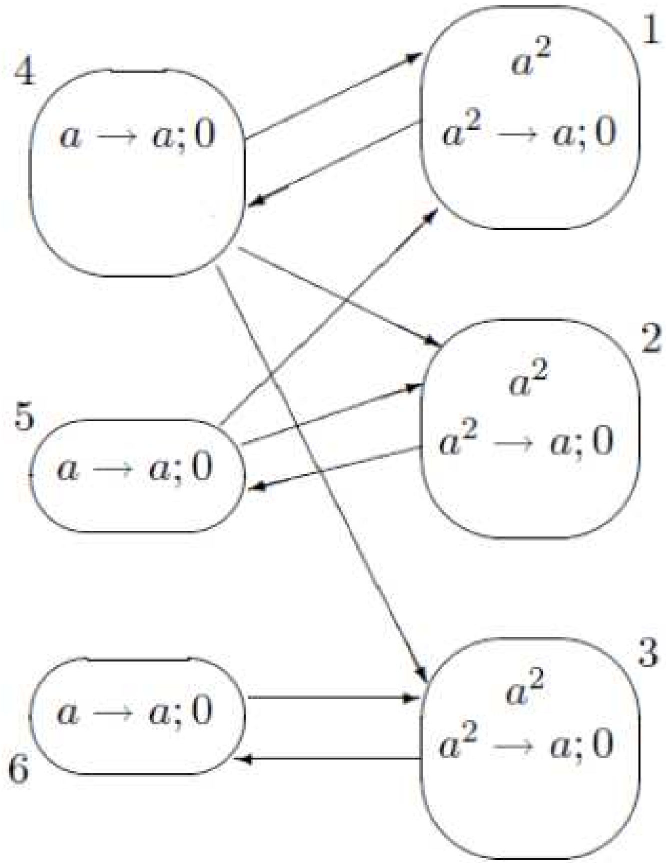
- Thus if the system $\boldsymbol{\pi}$ is periodic (implying that there exist a ***Null Aggregate Spiking Vector*** $\boldsymbol{(SV)_k^{k+v}}$), then Equation [5] above must have solutions other than the trivial one, which means that the determinant of the matrix M_π must be equal to zero.

■

Examples and Empirical Verifications

Example 1.
Recall the SN P system presented earlier:



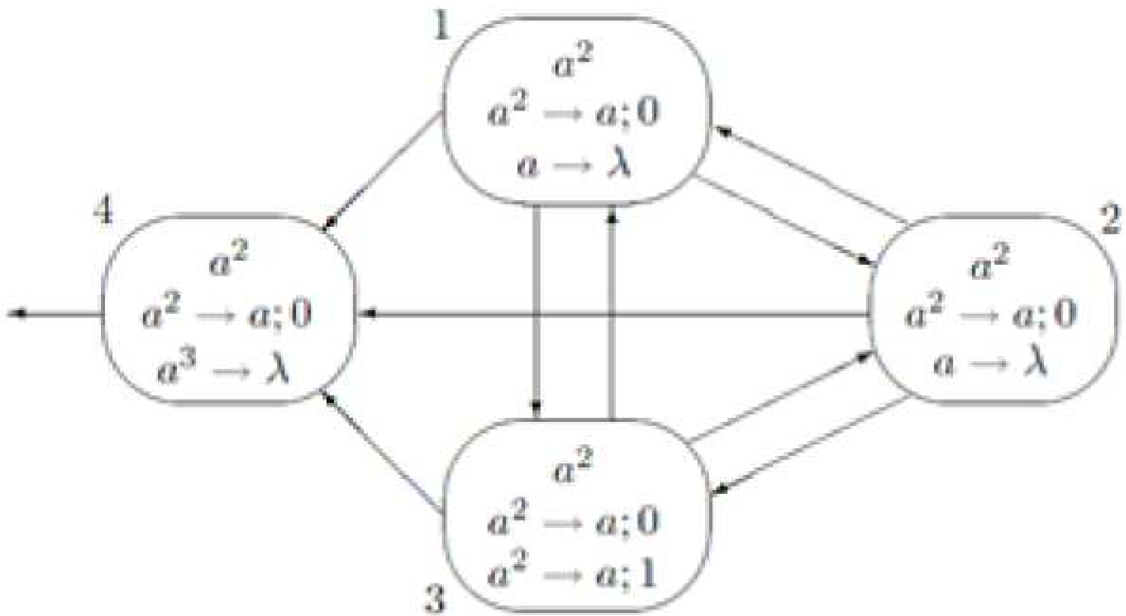


4
$a \to a; 0$
1
a^2
$a^2 \to a; 0$
5
$a \to a; 0$
2
a^2
$a^2 \to a; 0$
6
$a \to a; 0$
3
a^2
$a^2 \to a; 0$

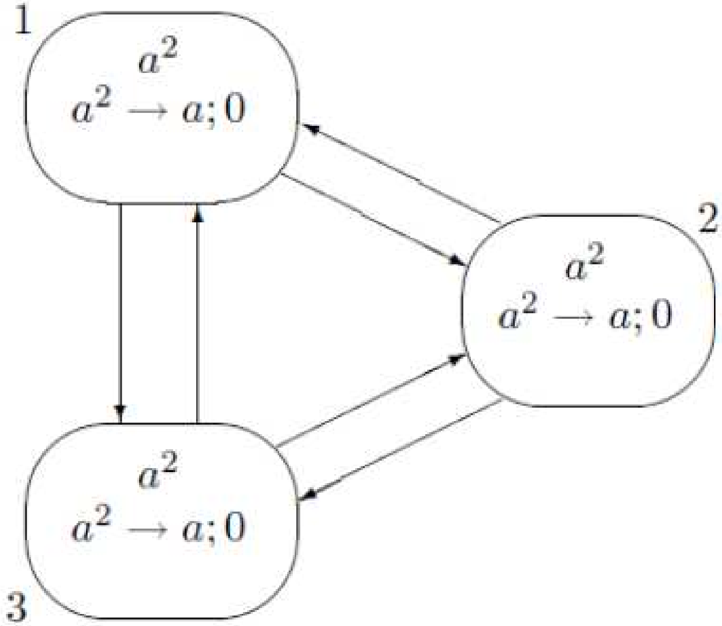
Spiking Transition Matrix of the SN P system above:

$$M_1 = \begin{bmatrix} -2 & 0 & 0 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 & 1 & 0 \\ 0 & 0 & -2 & 0 & 0 & 1 \\ 1 & 1 & 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix}$$

Example 2.



An SN P system generating all natural numbers.



$$M_3 = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

A deterministic, periodic SN P system,

with period = 1.

The process we followed in dissecting our examples above are the following:

- Take a nondeterministic SN P system that generates an infinite number of outputs.
- Remove some of its elements, e.g., the forgetting rules, the rules with delay, and the designated output neuron.
- What we obtained were deterministic, periodic SN P systems, with their periods corresponding to the interval of the numbers they generated. Moreover, the determinants of the Spiking Transition Matrices of these systems were empirically verified to be equal to zero.

Reversing such process:

- Start from a deterministic, periodic SN P system, with a particular period. The determinant of the Spiking Transition Matrix of this system has to be zero.

- Incorporate additional key components, e.g., forgetting rules, rules with delay, and a designated output neuron.

- What we will obtain is a nondeterministic SN P system that generates an infinite number of outputs, whose interval corresponds to the period of the core deterministic, periodic SN P system, from which it was built up.

A Systematic Design Framework for SN P systems:

- The reversed process just described above can be viewed as some sort of a systematic design framework for generative SN P systems.

- Given the specification of the property (i.e., interval) of the output one intends to generate, one can start from a deterministic, periodic SN P system, whose period correspond with the specified interval.

- The finishing touches are adding some particular elements to the system (e.g., forgetting rules, rules with delay, and a designated output neuron).

Final Remarks

- In this paper, we explored the study of periodicity in SN P systems as one of its dynamical aspects.

- We used matrix representation and other algebraic concepts, together with some of our additional definitions, to describe periodic properties in an SN P System.

- We adapted some examples from the literature of SN P systems and successfully showed some empirical verification, and we claimed that periodicity is an inherent property of generative SN P systems. We thus conjecture here that a finite generative SN P system generates an infinite number of outputs only if it is periodic.

- We believe that these algebraic representations, aside from being an elegant representation from a theoretical viewpoint, also paved the way for some sort of a systematic design framework.

On-going and Future Work

- How does an SN P system’s topology relates with its dynamical behavior?